Fon 2024 – detaljno rešeni zadaci sa prijemnog

1. ↑13000 ... ↑26%
   x дин ... 100%

$x : 13000 = 100 : 26$

$x = \frac{13000 \cdot 100}{26} = 50000$ дин цена л.т.

$50000 - (13000 + 13500) = 23500$ дин

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2. $\left(36{,}5+\left(\frac{1}{25}\right)^{-\frac{1}{2}}-\frac{3}{2}\right)^{\frac{1}{2}}\cdot\left(\sqrt[3]{27}+2^3-1\right)^{-\frac{1}{2}}=$

$\left(36{,}5+\sqrt{25}-\frac{3}{2}\right)^{\frac{1}{2}}\cdot\left(3+8-1\right)^{-\frac{1}{2}}=$

$\left(36{,}5+5-1{,}5\right)^{\frac{1}{2}}\cdot 10^{-\frac{1}{2}}=$

$40^{\frac{1}{2}}\cdot\frac{1}{10^{\frac{1}{2}}}=4^{\frac{1}{2}}=\sqrt{4}=2$

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3. $\left(\frac{ab}{a-b}+\overset{(a-b}{a}\right)\cdot\left(\frac{ab}{a+b}-\overset{(a+b}{a}\right):\frac{a^2b^2}{a^2-b^2}=$

$\frac{\cancel{ab}+a^2-\cancel{ab}}{a-b}\cdot\frac{\cancel{ab}-a^2-\cancel{ab}}{a+b}:\frac{a^2b^2}{a^2-b^2}=$

$\frac{-a^{\cancel{4}\,2}}{\cancel{(a-b)}\cancel{(a+b)}}\cdot\frac{\cancel{(a-b)}\cancel{(a+b)}}{\cancel{a^2}b^2}=-\frac{a^2}{b^2}$

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4. $(1+i)^{2024}=\left((1+i)^2\right)^{1012}=(2i)^{1012}=2^{1012}\cdot i^{1012}=2^{1012}$

$(1-i)^{2025}=(1-i)(1-i)^{2024}=(1-i)(-2i)^{1012}=2^{1012}\cdot \overset{1}{i^{1012}}\cdot(1-i)$

$=2^{1012}\cdot(1-i)$

$z=\frac{\cancel{2^{1012}}\cdot(1-i)}{\cancel{2^{1012}}}=1-i$

$Re(z)=1 \quad Im(z)=-1$

5.

$$f(x+3)=2x+5$$
$$x+3=t$$
$$x=t-3$$
$$f(t)=2(t-3)+5$$
$$f(t)=2t-6+5$$
$$f(t)=2t-1$$
$$f(x)=2x-1$$

$$g(f(x)-1)=3x+2$$
$$g(2x-1-1)=3x+2$$
$$g(2x-2)=3x+2$$
$$2x-2=t \Rightarrow x=\frac{t+2}{2}$$
$$g(t)=3\cdot\frac{t+2}{2}+2^{\,1\cdot 2}$$
$$g(t)=\frac{3t+10}{2}$$
$$g(x)=\frac{3x+10}{2}$$

$$h(x)=3f(x)+2g(x)$$
$$=3(2x-1)+2\cdot\frac{3x+10}{2}=6x-3+3x+10$$
$$=9x+7$$

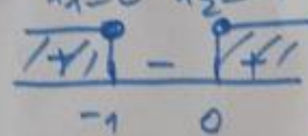
6.

$$1+9^{\sqrt{x^2+x}+0,5}=4\cdot 3^{\sqrt{x^2+x}}$$
$$1+9^{\sqrt{x^2+x}}\cdot 3=4\cdot 3^{\sqrt{x^2+x}}$$
$$3^{\sqrt{x^2+x}}=t$$
$$1+3t^2=4t$$
$$3t^2-4t+1=0$$
$$t_{1,2}=\frac{4\pm 2}{6}$$
$$t_1=1 \quad t_2=\frac{1}{3}=3^{-1}$$

$$3^{\sqrt{x^2+x}}=1=3^0$$
$$\sqrt{x^2+x}=0$$
$$x_1=-1 \quad x_2=0$$

$$\sqrt{x^2+x}=-1$$
немогуће
$$x_1+x_2=-1+0=-1$$

услов
$$x^2+x\geq 0$$
$$x(x+1)\geq 0$$
$$x_1=0 \quad x_2=-1$$



$$x\in(-\infty,-1]\cup[0,\infty)$$

7. $3^{\log_{\sqrt{5}}7\cdot\log_9 25} = 3^{\log_{5^{\frac{1}{2}}}7\cdot\log_{3^2}5^2}$ | $\log_A B^n = n\log_A B$

$= 3^{2\log_5 7\cdot\cancel{2}\cdot\frac{1}{\cancel{2}}\log_3 5}$ | $\log_{A^s} B = \frac{1}{s}\log_A B$

$= 3^{\log_5 49\cdot\frac{1}{\log_5 3}} = 3^{\log_3 49} = 49$ | $\log_A B = \frac{\log_c B}{\log_c A}$

$A^{\log_A B} = B$

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8. $\frac{x^2-12x+20}{x^2-7x+6} - 2 \geq 0$ $(x^2-7x+6$ $x^2-7x+6 \neq 0$

$x \neq 1 \wedge x \neq 6$

$\frac{x^2-12x+20-2x^2+14x-12}{x^2-7x+6} \geq 0$

$\frac{-x^2+2x+8}{x^2-7x+6} \geq 0 \quad /(-1)$

$\frac{x^2-2x-8}{x^2-7x+6} \leq 0 \qquad \frac{(x-4)(x+2)}{(x-1)(x-6)} \leq 0$

	$-\infty$ -2	1	4	6	∞
$x-4$	−	−	−	+	+
$x+2$	−	+	+	+	+
$x-1$	−	−	+	+	+
$x-6$	−	−	−	−	+
R	+	−	+	−	+

$x \in [-2, 1) \cup [4, 6)$

$-2, -1, 0$ и $4, 5 \rightarrow 5$ решения $\in \mathbb{Z}$

9. $b_1 + b_2 = 16$

$b_3 - b_1 = 32$

$b_1 + b_1 q = 16$

$b_1 q^2 - b_1 = 32$

$$: \begin{cases} b_1(1+q) = 16 \\ b_1(q^2-1) = 32 \end{cases}$$

$$\frac{\cancel{b_1}\,\cancel{(1+q)}}{\cancel{b_1}\,(q-1)\cancel{(q+1)}} = \frac{\cancel{16}^{\,1}}{\cancel{32}_{\,2}}$$

$$\frac{1}{q-1} = \frac{1}{2}$$

$$q-1 = 2 \Rightarrow \boxed{q=3}$$

$$b_1 \cdot (1+3) = 16$$

$$\boxed{b_1 = 4}$$

$4,\ 12,\ 36,\ 108$

$$b_1+b_2+b_3+b_4 = 4+12+36+108 = 160$$

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10. $\log_{x-1}(x+1) + 4\log_{x+1}(x-1) = 4$

<u>услови</u>

$x+1>0 \Rightarrow x>-1$

$x-1>0 \rightarrow x>1$

$x+1 \neq 1 \Rightarrow x \neq 0$

$x-1 \neq 1 \Rightarrow x \neq 2$

$x \in (1,2) \cup (2,\infty)$

$$\log_{x-1}(x+1) + \frac{4}{\log_{x-1}(x+1)} = 4$$

смена: $\log_{x-1}(x+1) = t$

$$t + \frac{4}{t} = 4 \quad /\cdot t$$

$$t^2 + 4 = 4t$$

$$t^2 - 4t + 4 = 0$$

$$(t-2)^2 = 0 \rightarrow t = 2$$

$$\log_{x-1}(x+1) = 2$$

$$x+1 = (x-1)^2$$

$$x+1 = x^2 - 2x + 1$$

$$0 = x^2 - 3x$$

$x_1 = 0$ не може $\quad \boxed{x_2 = 3}$ ✓

11. $Q(x) = x^3 + x = x(x^2+1) = x(x-i)(x+i)$

$x=0,\ x=i,\ x=-i$

$P(x) = Q(x) \cdot \text{нешто} + (Ax^2 + Bx + C)$

остатак

$P(0) = 0^{2024} + 0^{25} - 0^6 + 1 = 1$

$1 = 0 + A\cdot 0^2 + B\cdot 0 + C \Rightarrow \boxed{C=1}$

$P(i) = i^{2024} + i^{25} - i^6 + 1 = 1 + i + 1 + 1 = 3 + i$

$3 + i = A\cdot i^2 + Bi + 1 \Rightarrow 3 + i = -A + Bi + 1$

$P(-i) = (-i)^{2024} + (-i)^{25} - (-i)^6 + 1 = 1 - i + 1 + 1 = 3 - i$

$3 - i = A(-i)^2 + B\cdot(-i) + 1 \Rightarrow 3 - i = -A - Bi + 1$

$$\begin{cases} -A + Bi + 1 = 3 + i \Rightarrow B = 1 \\ -A - Bi + 1 = 3 - i \end{cases}$$

$+$

$-2A = 6 - 2$

$-2A = 4 \Rightarrow A = -2$

$Ax^2 + Bx + C = -2x^2 + x + 1$

остатак

12)

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ} =$$

адициона

$$= \frac{2\cdot\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{1}{2}\cdot \underbrace{2\sin 10^\circ \cdot \cos 10^\circ}_{\sin 2x}} = \frac{2\cdot\left(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ\right)}{\frac{1}{2}\sin 20^\circ}$$

$$= \frac{2\cdot \cancel{\sin(30^\circ - 10^\circ)}}{\frac{1}{2}\cancel{\sin 20^\circ}} = 4$$

13.

$$\sqrt{-x^2+x+6} > 1-x$$

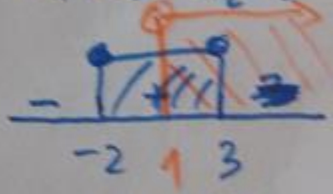
$$\sqrt{P} > Q \Leftrightarrow (P \geq 0 \wedge Q<0) \vee (P > Q^2 \wedge Q \geq 0)$$

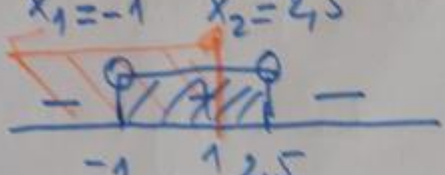
$$(-x^2+x+6 \geq 0 \wedge 1-x<0) \vee (-x^2+x+6 > (1-x)^2 \wedge 1-x \geq 0)$$

$-x^2+x+6=0$ $\quad$ $-x<-1$ $\quad$ | $\quad$ $-x^2+x+6>1-2x+x^2$ $\quad$ $x \leq 1$

$x_1=-2 \quad x_2=3$ $\quad$ $x>1$ $\quad$ | $\quad$ $-2x^2+3x+5>0$

$x_1=-1 \quad x_2=2{,}5$





$x \in (1, 3]$ $\qquad\qquad$ $x \in (-1, 1]$

конечно $\quad x \in (-1,1] \cup (1,3] \Rightarrow x \in (-1, 3]$

$x \in \{0,1,2,3\}$

14. $\frac{x^2}{4}-\frac{y^2}{3}=1 \qquad x^2+(y-7)^2=32$

За решимо прво сматем да нађемо пресек

$\frac{x^2}{4}=1+\frac{y^2}{3} \Rightarrow x^2=4\left(1+\frac{y^2}{3}\right)$

$x^2=4\cdot(1+3)$

$x_{1,2}=\pm4$

$4\left(1+\frac{y^2}{3}\right)+(y-7)^2=32$

$A=P_1(4,3) \quad P_2(-4,3)=B$

$4+\frac{4y^2}{3}+y^2-14y+49=32$

$\frac{7y^2}{3}-14y+21=0 \;/:7 \to \frac{y^2}{3}-2y+3=0$

$y^2-6y+9=0$

$(y-3)^2=0$

$\boxed{y=3}$

Сада тражимо заједничке тангенте

$\frac{x^2}{4}-\frac{y^2}{3}=1 \to a^2=4 \quad b^2=3$

из $x^2+(y-7)^2=32$

$p=0 \quad q=7 \quad r^2=32$

$a^2k^2-b^2=n^2$

$4k^2-3=n^2$

$r^2(k^2+1)=(kp-q+n)^2$

$k^2=\frac{n^2+3}{4} \to 32(k^2+1)=(-7+n)^2$

$k^2=\frac{(-1)^2+3}{4}=1 \qquad 32\left(\frac{n^2+3}{4}+1\right)=n^2-14n+49$

$k=\pm1 \qquad 8n^2+24+32-n^2+14n-49=0$

$7n^2+14n+7=0$

$t_1: y=x-1$

$n^2+2n+1=0$

$t_2: y=-x-1$

$(n+1)^2=0 \to \boxed{n=-1}$

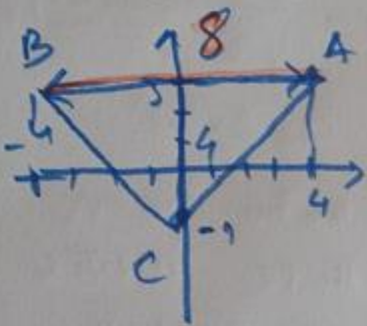
да нађемо C као $t_1 \cap t_2$ (систем)

$x-1=-x-1$

$x=0 \Rightarrow y=-1$ $\quad C(0,-1)$

имамо све три тачке $A(4,3)$ $B(-4,3)$

$C(0,-1)$



$$P=\frac{AB\cdot h_{AB}}{2}=\frac{8\cdot 4}{2}=16$$

а може и преко формуле за $P_\triangle$ или преко вектора (овако најпростије)

15.

$H = \frac{a\sqrt{6}}{3}$

$r = r_y$

$r_y = \frac{a\sqrt{3}}{6}$

$a\sqrt{3} = 6r$

$a = \frac{6r}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2r\sqrt{3}$

$a = 2\sqrt{3}r$

$V_v = r^2\pi H_v$

$V_T = \frac{1}{3} B H_T$

$V = \frac{1}{3} \frac{a^2\sqrt{3}}{4} \cdot \frac{a\sqrt{6}}{3}$

$V = \frac{\sqrt{2}a^3}{12}$ → неће треба него чисто да знамо

$V_v = V_T$

$r^2\pi H_v = \frac{1}{3} \frac{a^2\sqrt{3}}{4} \cdot H_T$

$\cancel{r^2}\pi H_v = \frac{1}{\cancel{12}} \cdot \cancel{(2\sqrt{3})^2} \cancel{r^2}\sqrt{3} \cdot H_T$

$\pi H_v = \sqrt{3} \cdot H_T$

$H_v : H_T = \sqrt{3} : \pi$

16.

$V = \frac{1}{3} B \cdot H$

$V = \frac{1}{3} r^2 \pi H$

$H = \frac{3V}{r^2\pi}$

$H^2 + r^2 = s^2$

$\frac{9V^2}{r^4\pi^2} + r^2 = s^2$

$\frac{9V^2 + r^6\pi^2}{r^4\pi^2} = s^2 \quad \rightarrow \quad s = \sqrt{\frac{9V^2 + r^6\pi^2}{r^4\pi^2}}$

$$M = sr\pi = \sqrt{\frac{9V^2 + r^6\pi^2}{r^4\pi^2}} \cdot r \cdot \pi = \sqrt{\frac{9V^2 + r^6\pi^2}{r^2\pi^2}} \cdot \pi$$

$$M'_r = \frac{\pi}{2\sqrt{\frac{9V^2 + r^6\pi^2}{r^2\pi^2}}} \cdot \frac{6\pi^2 r^5 \cdot r^2\pi^2 - 2r\pi^2(9V^2 + r^6\pi^2)}{(r^2\pi^2)^2} = 0$$

$$6\pi^4 r^7 - 18rV^2\pi^2 - 2r^7\pi^4 = 0$$

$$4r^7\pi^4 = 18rV^2\pi^2$$

$$r^6 = \frac{9V^2}{2\pi^2} \Rightarrow r = \sqrt[6]{\frac{9V^2}{2\pi^2}}$$

$$H = \frac{3V}{r^2\pi} = \frac{3V}{\pi} \cdot \sqrt[3]{\frac{2\pi^2}{9V^2}} = \sqrt[3]{\frac{2\pi^2}{9V^2} \cdot \frac{27V^3}{\pi^3}}$$

$$H = \sqrt[3]{\frac{6V}{\pi}}$$

сад тражимо s

$$s^2 = \sqrt[3]{\frac{9v^2}{2\pi^2}} + \sqrt[3]{\frac{72v^2}{2\pi^2}}$$

$$s^2 = \frac{\sqrt[3]{9v^2} + 2\sqrt[3]{9v^2}}{\sqrt[3]{2\pi^2}} = \frac{3\sqrt[3]{9v^2}}{\sqrt[3]{2\pi^2}}$$

$$s = \sqrt{3}\sqrt[6]{\frac{9v^2}{2\pi^2}}$$

$$M = s r \pi$$

$$M = \sqrt{3}\sqrt[6]{\frac{9v^2}{2\pi^2}} \cdot \sqrt[6]{\frac{9v^2}{2\pi^2}} \cdot \pi$$

$$M = \sqrt{3}\sqrt[6]{\frac{81v^4}{4\pi^4}} \cdot \sqrt[6]{\pi^6}$$

$$M = \sqrt{3}\sqrt[6]{\frac{81v^4}{4\pi^4}\cdot\pi^6} = \sqrt{3}\sqrt[6]{\frac{81v^4\pi^2}{4}}$$

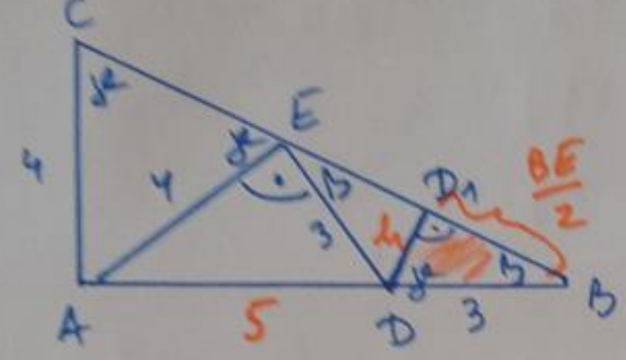
$$M = \sqrt{3}\sqrt[3]{\frac{9v^2\pi}{2}}$$

$$\sqrt[3]{9} = \sqrt[3]{\sqrt{3}^4} = \sqrt[3]{\sqrt{3}^3\cdot\sqrt{3}} = \sqrt{3}\sqrt[3]{\sqrt{3}}$$

$$M = \sqrt{3}\cdot\sqrt{3}\sqrt[3]{\frac{\sqrt{3}v^2\pi}{2}}$$

$$\boxed{M = 3\sqrt[3]{\frac{\sqrt{3}\cdot v^2\pi}{2}}}$$

17.



$\triangle ABE$ је правоугли

$AD^2 = 4^2 + 3^2 = 16 + 9 = 25$

$AD = 5$ cm

$\triangle DD_1B \sim \triangle ABC$

$4 : h_1 = 4\sqrt{5} : 3$

$h_1 = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

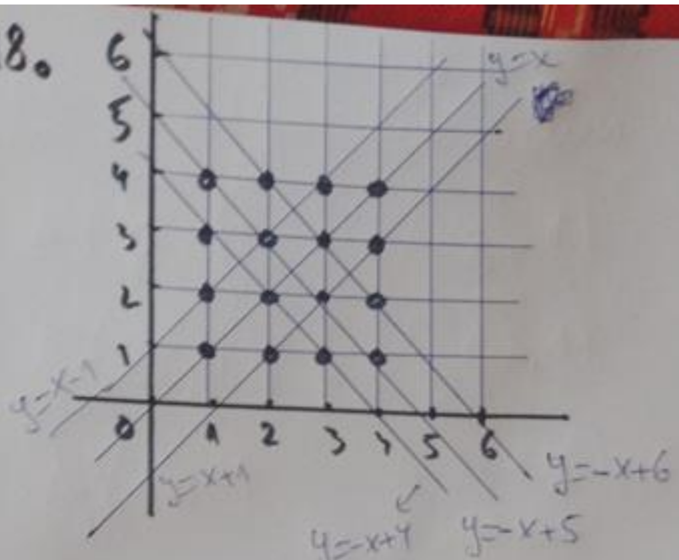
$8 : \frac{BE}{2} = 4\sqrt{5} : 3$

$BE = \frac{12}{\sqrt{5}} = \frac{12\sqrt{5}}{5}$

$P_{\triangle BDD_1} = \frac{1}{2} BE \cdot h_1 = \frac{1}{\cancel{2}} \cdot \frac{3\sqrt{5}}{5} \cdot \frac{\cancel{12}^{6}\sqrt{5}}{5} = \frac{18}{5}$

$= 3{,}6\ cm^2$

18.



имамо 16 тачака, од њих бирамо 3

$C^{16}_{3} = \frac{16\cdot15\cdot14}{3\cdot2\cdot1} = 560$

Сад морамо да избацимо:

→ ако све тачке имају исту x-коор.

нпр. $\{A_1(1,1)\ A_2(1,2)\ A_3(1,3)$ за $x=1$ → има их 4

за $x=2$ → –||– 4

за $x=3$ → –||– 4

за $x=4$ → –||– 4

16

→ ако све тачке имају исту y-координату исто бацамо 16

→ на правама $y=x$ и $y=-x+5$ бацамо по 4 то је 8

→ на правама $y=x+1$, $y=x-1$, $y=-x+4$, $y=-x+6$ бацамо по 1 → значи 4

укупно бацамо $16+16+12=44$ $\quad 560-44=\boxed{516}$

19. $\left(\sqrt{6}+\frac{1}{\sqrt[3]{2}}\right)^{40} \rightarrow a=\sqrt{6}=6^{\frac{1}{2}}$

$T_{k+1}=\binom{n}{k}a^{n-k}b^{k}$ $\quad b=\frac{1}{\sqrt[3]{2}}=\frac{1}{2^{\frac{1}{3}}}$ $\quad n=40$

$T_{k+1}=\binom{40}{k}\left(6^{\frac{1}{2}}\right)^{40-k}\cdot\left(\frac{1}{2^{\frac{1}{3}}}\right)^{k}$

$T_{k+1}=\binom{40}{k}\frac{6^{\frac{40-k}{2}}}{2^{\frac{k}{3}}}$ $\quad k=0,1,2,\dots,40$

$\frac{40-k}{2}$ и $\frac{k}{3}$ морају бити цели и још водимо рачуна да $\frac{40-k}{2} \geq \frac{k}{3}$ да би могли да се скрате да не остане 2 доле (биће $\mathbb{Q}$)

$k=0 \rightarrow \frac{40-k}{2}=20 \quad \frac{k}{3}=0 \rightarrow$ ради

$k=3 \rightarrow \frac{40-3}{2}=\frac{37}{2} \rightarrow$ неће

$k=6 \rightarrow \frac{40-6}{2}=17 \quad \frac{k}{3}=\frac{6}{3}=2$ ✓

$k=12 \rightarrow \frac{40-12}{2}=14 \quad \frac{12}{3}=4$ ✓

$k=18 \rightarrow \frac{40-18}{2}=11 \quad \frac{18}{3}=6$ ✓ 5

$k=24 \rightarrow \frac{40-24}{2}=8 \quad \frac{24}{3}=8$ ✓ их има

$k=30 \rightarrow \frac{40-30}{2}=5 \quad \frac{30}{3}=10 \rightarrow$ неће

20. $11\cos 2x - 3 = 3\sin 3x - 11\sin x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$

$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= -4\sin^3 x + 3\sin x$

$11(1 - 2\sin^2 x) - 3 = 3(-4\sin^3 x + 3\sin x) - 11\sin x$

$11 - 22\sin^2 x - 3 = -12\sin^3 x - 2\sin x$

$12\sin^3 x - 22\sin^2 x + 2\sin x + 8 = 0 \;/:2$

$6\sin^3 x - 11\sin^2 x + \sin x + 4 = 0$

$\sin x = t$

$6t^3 - 11t^2 + t + 4 = 0$

Безуова теорем

$+1 \quad +2 \quad +4$

$-1 \quad -2 \quad -4$

пробамо

за $t = 1 \rightarrow 6 - 11 + 1 + 4 = 0$

ради

ш

сад делимо полиноме

$$(6t^3 - 11t^2 + t + 4) : (t - 1) = 6t^2 - 5t - 4$$

$$\ominus 6t^3 \overset{-}{\oplus} 6t^2$$

$$-5t^2 + t$$

$$\overset{-}{\oplus} 5t^2 \overset{+}{\ominus} 5t$$

$$-4t + 4$$

$$\overset{-}{\oplus} 4t \overset{+}{\ominus} 4$$

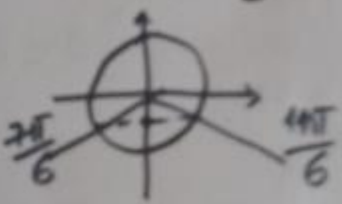
$$6t^2 - 5t - 4 = 0$$

$$t_1 = -\frac{1}{2} \quad t_2 = \frac{4}{3}$$

$\sin x = 1 \;\vee\; \sin x = -\frac{1}{2} \;\vee\; \sin x = \frac{4}{3}$ nemoguće

$x = \frac{\pi}{2} + 2k\pi$

$[0, 2\pi)$



za $k=0$ $x = \frac{\pi}{2}$

za $k=1$ $\cancel{x = 2\pi + \frac{\pi}{2}}$

$x = \frac{7\pi}{6} + 2k\pi \;\vee\; x = \frac{11\pi}{6} + 2k\pi$

za $k=1 \rightarrow x = \frac{7\pi}{6}$ za $k=1$ $x = \frac{11\pi}{6}$

$$\frac{\pi}{2} + \frac{7\pi}{6} + \frac{11\pi}{6} = \frac{3\pi + 18\pi}{6}$$

$$= \frac{21\pi}{6} = \frac{7\pi}{2}$$