

R E Š E N J A

Klasifikacionog ispita iz Matematike za 2016. godinu:

1. Uprostiti izraz $I = \frac{x^2 - 4xy + 3y^2}{x^2 - y^2}$.

$$\begin{aligned} I &= \frac{x^2 - 4xy + 3y^2}{x^2 - y^2} = \frac{x^2 - xy - 3xy + 3y^2}{(x - y)(x + y)} = \frac{x(x - y) - 3y(x - y)}{(x - y)(x + y)} = \\ &= \frac{(x - y)(x - 3y)}{(x - y)(x + y)} = \boxed{\frac{x - 3y}{x + y}}, \text{ za } x \neq \pm y. \end{aligned}$$

2. Rastaviti na faktore polinom $P(x) = x^5 - x^3 - x^2 + 1$.

$$\begin{aligned} P(x) &= x^3(x^2 - 1) - (x^2 - 1) = (x^2 - 1)(x^3 - 1) \\ &= (x - 1)(x + 1)(x - 1)(x^2 + x + 1) = \boxed{(x - 1)^2(x + 1)(x^2 + x + 1)}. \end{aligned}$$

3. Uprostiti izraz $I = \frac{1 + x + (1 - x)^{-1}}{1 + (1 - x^2)^{-1}}$.

$$\begin{aligned} I &= \frac{1 + x + (1 - x)^{-1}}{1 + (1 - x^2)^{-1}} = \frac{1 + x + \frac{1}{1 - x}}{1 + \frac{1}{1 - x^2}} = \frac{\frac{1 - x + x - x^2 + 1}{1 - x}}{\frac{1 - x^2 + 1}{(1 - x)(1 + x)}} = \\ &= \frac{(2 - x^2)(1 - x)(1 + x)}{(2 - x^2)(1 - x)} = \boxed{1 + x}, \text{ za } x \neq \pm 1, x \neq \pm\sqrt{2}. \end{aligned}$$

4. Rešiti jednačinu $\frac{9x - 8}{7} = 7 - \frac{5x + 7}{9}$.

Ako jednačinu pomnožimo sa 63 dobijamo

$$\begin{aligned} 9(9x - 8) &= 7 \cdot 63 - 7(5x + 7) \iff 81x - 72 = 441 - 35x - 49 \iff \\ 116x &= 464 \iff \boxed{x = 4}. \end{aligned}$$

5. Rešiti jednačinu $\frac{x^2 - 2x}{x^2 - 4} = 0$.

$$\begin{aligned} \text{Za } x \neq \pm 2 \text{ je } x^2 - 2x = 0 &\iff x(x - 2) = 0 \iff x = 0 \vee x - 2 = 0 \\ &\iff \boxed{x = 0} \end{aligned}$$

6. Rešiti sistem jednačina $3x + 5y = 1 \wedge 3x - 2y = 8$.

Ako prvu jednačinu pomnožimo sa -1 i dodamo drugoj, dobijamo

$$\begin{aligned} -3x - 5y = -1 \wedge 3x - 2y = 8 &\iff -7y = 7 \wedge 3x - 2y = 8 \\ &\iff y = -1 \wedge 3x + 2 = 8 \iff \boxed{x = 2} \wedge \boxed{y = -1}. \end{aligned}$$

7. Rešiti jednačinu $(x + 1)^2 - 25 = 0$.

$$(x + 1)^2 - 25 = 0 \iff (x + 1)^2 - 5^2 = 0 \iff (x + 1 - 5)(x + 1 + 5) = 0 \iff \\ (x - 4)(x + 6) = 0 \iff x - 4 = 0 \vee x + 6 = 0 \iff \boxed{x = 4} \vee \boxed{x = -6}.$$

8. Za koju vrednost parametra $m \in \mathbb{R}$ kvadratna jednačina $x^2 + 6x + m = 0$ ima realna rešenja?

Jednačina ima realna rešenja ako i samo ako je $D = b^2 - 4ac \geq 0$, tj. ako je

$$D = 6^2 - 4 \cdot 1 \cdot m \geq 0 \iff 36 - 4m \geq 0 \iff \boxed{m \leq 9}.$$

9. Rešiti nejednačinu $(x - 2)^2 - 9 > 0$.

$$(x - 2)^2 - 3^2 = (x + 1)(x - 5) > 0 \iff \\ [(x + 1 < 0 \wedge x - 5 < 0) \vee (x + 1 > 0 \wedge x - 5 > 0)] \iff \\ [x < -1 \vee x > 5] \iff \boxed{x \in (-\infty, -1) \cup (5, +\infty)}.$$

10. Rešiti jednačinu $(2016)^{x^2 - 5x + 4} = 1$.

$$(2016)^{x^2 - 5x + 4} = 1 \iff (2016)^{x^2 - 5x + 4} = (2016)^0 \iff x^2 - 5x + 4 = 0 \iff \\ \boxed{x = 1} \vee \boxed{x = 4}.$$

11. Rešiti jednačinu $\log_4(3x + 4) = 3$.

Za $3x + 4 > 0 \iff x > -\frac{4}{3}$ je

$$\log_4(3x + 4) = 3 \iff 3x + 4 = 4^3 \iff 3x = 60 \iff \boxed{x = 20}.$$

12. Izračunati vrednost izraza $I = \log_6 2 + \log_6 3$.

$$I = \log_6 2 + \log_6 3 = \log_6(2 \cdot 3) = \log_6 6 = \boxed{1}.$$

13. Rešiti nejednačinu $\frac{x - 1}{x + 2} < 1$.

$$\text{Za } x \neq -2, \text{ je } \frac{x - 1}{x + 2} < 1 \iff \frac{x - 1}{x + 2} - 1 < 0 \iff \frac{x - 1 - x - 2}{x + 2} < 0 \iff \frac{-3}{x + 2} < 0 \\ \iff x + 2 > 0 \iff x > -2 \iff \boxed{x \in (-2, +\infty)}.$$

14. Rešiti jednačinu $4^x - 5 \cdot 2^x + 4 = 0$.

$$4^x - 5 \cdot 2^x + 4 = 0 \iff 2^{2x} - 5 \cdot 2^x + 4 = 0 \iff (2^x)^2 - 5 \cdot 2^x + 4 = 0.$$

Smenom $2^x = t$ dobijamo: $t^2 - 5t + 4 = 0 \iff t = 1 \vee t = 4$, pa je:

$$4^x - 5 \cdot 2^x + 4 = 0 \iff 2^x = 1 \vee 2^x = 4 \iff \boxed{x = 0} \vee \boxed{x = 2}.$$

15. Napisati kanonski oblik parabole $y = x^2 - 4x + 3$.

$$y = a \left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} = 1 \cdot \left(x + \frac{-4}{2 \cdot 1} \right)^2 + \frac{4 \cdot 1 \cdot 3 - (-4)^2}{4 \cdot 1} = \boxed{(x - 2)^2 - 1}.$$

Ispitivač: Prof. dr Žarko Popović