Stepenovanje (krugova zbirka)

Tražili ste od nas da odradimo nekoliko primera iz Krug-ove zbirke.

Naš savet Vam je da najpre obnovite iz I godine postupke za rastavljanje na činioce i operacije sa racionalnim algebarskim izrazima, pa onda krenite sa vežbanjem ovih primera sa stepenovanjem.

Evo nekoliko malo težih primera...

Primer 1.

Uprosti izraz $\frac{a^{-2}+b^{-2}}{a^{-1}+b^{-1}}\cdot\left(\frac{a^2+b^2}{ab}\right)^{-1}:\frac{a^{-1}-b^{-1}}{a^2-b^2}=$

Rešenje:

$$\frac{a^{-2}+b^{-2}}{a^{-1}+b^{-1}}\cdot\left(\frac{a^2+b^2}{ab}\right)^{-1}:\frac{a^{-1}-b^{-1}}{a^2-b^2}=$$

$$\frac{\frac{1}{a^2}+\frac{1}{b^2}}{\frac{1}{a}+\frac{1}{b}}\cdot\frac{ab}{a^2+b^2}:\frac{\frac{1}{a}-\frac{1}{b}}{a^2-b^2}=$$

$$\frac{\frac{b^2+a^2}{a^2b^2}}{\frac{b+a}{ab}}\cdot\frac{ab}{a^2+b^2}:\frac{\frac{b-a}{ab}}{a^2-b^2}=$$

$$\frac{(a^2+b^2)ab}{a^2b^2(a+b)}\cdot\frac{ab}{a^2+b^2}:\frac{b-a}{ab(a^2-b^2)}=\text{ pokratimo sve šta ima....}$$

$$\frac{1}{a+b}:\frac{b-a}{ab(a^2-b^2)}=$$

$$\frac{1}{a+b}\cdot\frac{ab(a-b)(a+b)}{-(a-b)}=\text{ pokratimo...}=-ab$$

Primer 2.

Uprosti izraz $\left(\frac{x^{-1}+y^{-1}}{yx^{-1}+xy^{-1}}\right)^{-1}+\left(\frac{x^{-1}+y^{-1}}{2}\right)^{-1}-\frac{x^{-1}-y^{-1}}{x^{-1}y^{-1}}=$

Rešenje:

$$\left(\frac{x^{-1}+y^{-1}}{yx^{-1}+xy^{-1}}\right)^{-1}+\left(\frac{x^{-1}+y^{-1}}{2}\right)^{-1}-\frac{x^{-1}-y^{-1}}{x^{-1}y^{-1}}=$$

$$\left(\frac{\frac{1}{x}+\frac{1}{y}}{\frac{y}{x}+\frac{x}{y}}\right)^{-1}+\left(\frac{\frac{1}{x}+\frac{1}{y}}{2}\right)^{-1}-\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{xy}}=$$

$$\left(\frac{\frac{y+x}{xy}}{\frac{y^2+x^2}{xy}}\right)^{-1}+\left(\frac{\frac{y+x}{xy}}{2}\right)^{-1}-\frac{\frac{y-x}{xy}}{\frac{1}{xy}}=$$

$$\left(\frac{xy(x+y)}{xy(x^2+y^2)}\right)^{-1}+\left(\frac{x+y}{2xy}\right)^{-1}-\frac{xy(y-x)}{xy}=\text{pokratimo xy a ovaj na -1 okrenemo....}$$

$$\frac{x^2+y^2}{x+y}+\frac{2xy}{x+y}-\frac{(y-x)}{1}=\text{zajednički je } (x+y)$$

$$\frac{x^2+y^2+2xy-(y-x)(x+y)}{x+y}=\text{izvučemo minus iz } y-x=-(x-y)$$

$$\frac{x^2+y^2+2xy+(x-y)(x+y)}{x+y}=$$

$$\frac{x^2+y^2+2xy+x^2-y^2}{x+y}=\text{potiremo } y^2$$

$$\frac{2x^2+2xy}{x+y}=\frac{2x(x+y)}{x+y}=2x$$

Primer 3.

Uprosti izraz $\dfrac{\left(ab^{-1}+1\right)^2}{ab^{-1}-a^{-1}b}\cdot\dfrac{a^3b^{-3}-1}{a^2b^{-2}+ab^{-1}+1}:\dfrac{a^3b^{-3}+1}{ab^{-1}+a^{-1}b-1}=$

Rešenje:

$$\frac{\left(ab^{-1}+1\right)^2}{ab^{-1}-a^{-1}b}\cdot\frac{a^3b^{-3}-1}{a^2b^{-2}+ab^{-1}+1}:\frac{a^3b^{-3}+1}{ab^{-1}+a^{-1}b-1}=$$

$$\frac{\left(\frac{a}{b}+1\right)^2}{\frac{a}{b}-\frac{b}{a}}\cdot\frac{\frac{a^3}{b^3}-1}{\frac{a^2}{b^2}+\frac{a}{b}+1}:\frac{\frac{a^3}{b^3}+1}{\frac{a}{b}+\frac{b}{a}-1}=$$

Radićemo prvo „direktno“ a posle ćemo pokazati da možemo uzeti smenu i malo sebi olakšati posao....

I način (direktno)

$$\frac{\left(\frac{a}{b}+1\right)^2}{\frac{a}{b}-\frac{b}{a}}\cdot\frac{\frac{a^3}{b^3}-1}{\frac{a^2}{b^2}+\frac{a}{b}+1}:\frac{\frac{a^3}{b^3}+1}{\frac{a}{b}+\frac{b}{a}-1}$$

$$\frac{\left(\frac{a+b}{b}\right)^2}{\frac{a^2-b^2}{ab}}\cdot\frac{\frac{a^3-b^3}{b^3}}{\frac{a^2+ab+b^2}{b^2}}:\frac{\frac{a^3+b^3}{b^3}}{\frac{a^2+b^2-ab}{ab}}=$$

$$\frac{(a+b)^2}{\frac{a^2-b^2}{ab}}\cdot\frac{\frac{(a-b)(a^2+ab+b^2)}{b^3}}{\frac{a^2+ab+b^2}{b^2}}:\frac{\frac{(a+b)(a^2-ab+b^2)}{b^3}}{\frac{a^2+b^2-ab}{ab}}=$$

$$\frac{(a+b)^2ab}{(a^2-b^2)\mathrm{b}^2}\cdot\frac{(a-b)(a^2+ab+b^2)b^2}{(a^2+ab+b^2)\mathrm{b}^3}:\frac{ab(a+b)(a^2+b^2-ab)}{\mathrm{b}^3(a^2+b^2-ab)}=\text{pokratimo sve šta ima...}$$

$$\frac{(a+b)^2a}{(a^2-b^2)b}\cdot\frac{(a-b)}{\mathrm{b}}:\frac{a(a+b)}{\mathrm{b}^2}=\text{okrenemo zadnji...}$$

$$\frac{(a+b)^2a}{(a^2-b^2)b}\cdot\frac{(a-b)}{\mathrm{b}}\cdot\frac{b^2}{a(a+b)}=\text{opet pokratimo}$$

$$\frac{(a+b)(a-b)}{(a^2-b^2)}=1$$

II način (uzimamo smenu $\frac{a}{b}=t\rightarrow\frac{b}{a}=\frac{1}{t}$)

$$\frac{\left(\frac{a}{b}+1\right)^2}{\frac{a}{b}-\frac{b}{a}}\cdot\frac{\frac{a^3}{b^3}-1}{\frac{a^2}{b^2}+\frac{a}{b}+1}:\frac{\frac{a^3}{b^3}+1}{\frac{a}{b}+\frac{b}{a}-1}=$$

$$\frac{(t+1)^2}{t-\frac{1}{t}}\cdot\frac{t^3-1}{t^2+t+1}:\frac{t^3+1}{t+\frac{1}{t}-1}=$$

$$\frac{(t+1)^2}{\frac{t^2-1}{t}}\cdot\frac{(t-1)(t^2+t+1)}{t^2+t+1}:\frac{(t+1)(t^2-t+1)}{\frac{t^2+1-t}{t}}=\text{pokratimo šta može}$$

$$\frac{t(t+1)^2}{t^2-1}\cdot(t-1):\frac{t(t+1)}{1}=\text{okrenemo zadnji}$$

$$\frac{t(t+1)^2}{t^2-1}\cdot(t-1)\cdot\frac{1}{t(t+1)}=\text{pokratimo}=\frac{(t-1)(t+1)}{t^2-1}=1$$

Primer 4.

Uprosti izraz $\dfrac{1-x^{-4}}{x-x^{-1}}-\dfrac{2}{x^3}+\dfrac{x^{-4}-x^2}{x-x^{-1}}$

Rešenje:

$$\frac{1-x^{-4}}{x-x^{-1}}-\frac{2}{x^3}+\frac{x^{-4}-x^2}{x-x^{-1}}=$$

$$\frac{1-\dfrac{1}{x^4}}{x-\dfrac{1}{x}}-\frac{2}{x^3}+\frac{\dfrac{1}{x^4}-x^2}{x-\dfrac{1}{x}}=$$

$$\frac{\dfrac{x^4-1}{x^4}}{\dfrac{x^2-1}{x}}-\frac{2}{x^3}+\frac{\dfrac{1-x^6}{x^4}}{\dfrac{x^2-1}{x}}=$$

Ovo x^4-1 ćemo tretirati kao razliku kvadrata $\left(x^2\right)^2-1^2=\left(x^2-1\right)\left(x^2+1\right)$

$1-x^6=-(x^6-1)=-((x^2)^3-1^3)\rightarrow$ kao razlika kubova

$$=-\left(x^2-1\right)\left(x^4+x^2+1\right)$$

$$\frac{\dfrac{\left(x^2-1\right)\left(x^2+1\right)}{x^4}}{\dfrac{x^2-1}{x}}-\frac{2}{x^3}+\frac{\dfrac{-\left(x^2-1\right)\left(x^4+x^2+1\right)}{x^4}}{\dfrac{x^2-1}{x}}=$$

$$\frac{x\left(x^2-1\right)\left(x^2+1\right)}{x^4\left(x^2-1\right)}-\frac{2}{x^3}-\frac{x\left(x^2-1\right)\left(x^4+x^2+1\right)}{x^4\left(x^2-1\right)}=\text{pokratimo sve što može}$$

$$\frac{x^2+1}{x^3}-\frac{2}{x^3}-\frac{x^4+x^2+1}{x^3}=\frac{x^2+1-2-x^4-x^2-1}{x^3}=\frac{-x^4-2}{x^3}$$

Primer 5.

Uprosti izraz $\dfrac{x^{-6}-64}{4+2x^{-1}+x^{-2}}\cdot\dfrac{x^2}{4-4x^{-1}+x^{-2}}-\dfrac{4x^2(2x+1)}{1-2x}=$

Rešenje:

$$\frac{x^{-6}-64}{4+2x^{-1}+x^{-2}}\cdot\frac{x^2}{4-4x^{-1}+x^{-2}}-\frac{4x^2(2x+1)}{1-2x}=$$

$$\frac{\frac{1}{x^6}-64}{4+\frac{2}{x}+\frac{1}{x^2}}\cdot\frac{x^2}{4-\frac{4}{x}+\frac{1}{x^2}}-\frac{4x^2(2x+1)}{1-2x}=$$

$$\frac{\frac{1-64x^6}{x^6}}{\frac{4x^2+2x+1}{x^2}}\cdot\frac{x^2}{\frac{4x^2-4x+1}{x^2}}-\frac{4x^2(2x+1)}{1-2x}=$$

$1-64x^6=1^3-\left(4x^2\right)^3$ = sad kao razlika kubova $=(1-4x^2)(1+4x^2+16x^4)$

$$\frac{x^2(1-4x^2)(1+4x^2+16x^4)}{x^6\left(4x^2+2x+1\right)}\cdot\frac{x^4}{\left(4x^2-4x+1\right)}-\frac{4x^2(2x+1)}{1-2x}=\text{pokratimo}=$$

$$\frac{\overset{\text{razlika kvadrata}}{(1-4x^2)}\,(1+4x^2+16x^4)}{\left(4x^2+2x+1\right)\underset{\text{pun kvadrat}}{\left(4x^2-4x+1\right)}}-\frac{4x^2(2x+1)}{1-2x}=$$

$$\frac{(1-2x)(1+2x)(1+4x^2+16x^4)}{\left(4x^2+2x+1\right)(1-2x)^2}-\frac{4x^2(2x+1)}{1-2x}=\text{pokratimo}=$$

$$\frac{(1+2x)(1+4x^2+16x^4)}{\left(4x^2+2x+1\right)(1-2x)}-\frac{4x^2(2x+1)}{1-2x}=$$

$$\frac{(1+2x)(1+4x^2+16x^4)-4x^2(2x+1)\left(4x^2+2x+1\right)}{\left(4x^2+2x+1\right)(1-2x)}=\text{ izvučemo (1+2x) kao zajednički}=$$

$$\frac{(1+2x)\left((1+4x^2+16x^4)-4x^2\left(4x^2+2x+1\right)\right)}{\left(4x^2+2x+1\right)(1-2x)}=\frac{(1+2x)\left(1+4x^2+16x^4-16x^4-8x^3-4x^2\right)}{\left(4x^2+2x+1\right)(1-2x)}=$$

$$\frac{(1+2x)\,\overset{\text{razlika kubova}}{(1-8x^3)}}{\left(4x^2+2x+1\right)(1-2x)}=\frac{(1+2x)(1-2x)(1+2x+4x^2)}{\left(4x^2+2x+1\right)(1-2x)}=1+2x$$

Primer 6.

Uprosti izraz $\dfrac{a^{-1}-(b+c)^{-1}}{a^{-1}+(b+c)^{-1}}\cdot(1+\dfrac{b^2+c^2-a^2}{2bc}):(\dfrac{abc}{a-b-c})^{-1}$

Rešenje:

$$\frac{a^{-1}-(b+c)^{-1}}{a^{-1}+(b+c)^{-1}}\cdot(1+\frac{b^2+c^2-a^2}{2bc}):(\frac{abc}{a-b-c})^{-1}=$$

$$\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\frac{\overbrace{2bc+b^2+c^2-a^2}^{\text{ova tri daju pun kvadrat}}}{2bc}:\frac{a-b-c}{abc}=$$

$$\frac{\frac{b+c-a}{a(b+c)}}{\frac{b+c+a}{a(b+c)}}\cdot\frac{\overbrace{(b+c)^2-a^2}^{\text{ovo sad kao razlika kvadrata}}}{2bc}\cdot\frac{abc}{a-b-c}=$$

$$\frac{b+c-a}{b+c+a}\cdot\frac{(b+c-a)(b+c+a)}{2bc}\cdot\frac{abc}{-(b+c-a)}=\text{pokratimo sve šta može}$$

$$=\frac{-a(b+c-a)}{2}=\text{izvučemo minus iz zagrade}=\frac{a(a-b-c)}{2}$$

Primer 7.

Uprosti izraz $\dfrac{a^{-1}-b^{-1}}{a^{-3}+b^{-3}}:\dfrac{a^2b^2}{(a+b)^2-3ab}\cdot\left(\dfrac{a^2-b^2}{ab}\right)^{-1}$

Rešenje:

$$\frac{a^{-1}-b^{-1}}{a^{-3}+b^{-3}}:\frac{a^2b^2}{(a+b)^2-3ab}\cdot\left(\frac{a^2-b^2}{ab}\right)^{-1}=$$

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}}:\frac{a^2b^2}{a^2+2ab+b^2-3ab}\cdot\frac{ab}{a^2-b^2}=$$

$$\frac{\frac{b-a}{ab}}{\frac{b^3+a^3}{a^3b^3}}\cdot\frac{a^2-ab+b^2}{a^2b^2}\cdot\frac{ab}{a^2-b^2}=$$

$$\frac{a^3b^3(b-a)}{ab(b+a)(b^2-ab+a^2)}\cdot\frac{a^2-ab+b^2}{a^2b^2}\cdot\frac{ab}{(a-b)(a+b)}=\text{pokratimo}$$

$$\frac{ab(b-a)}{(a+b)^2(a-b)}=\frac{-ab(a-b)}{(a+b)^2(a-b)}=\frac{-ab}{(a+b)^2}$$