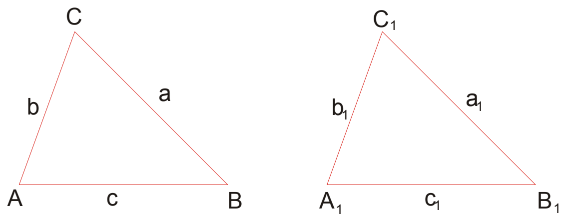
PODUDARNOST TROUGLOVA

Po definiciji, dva trougla $\triangle ABC$ i $\triangle A_1B_1C_1$ su podudarni ako postoji izometrija koja $\triangle ABC$ prevodi u $\triangle A_1B_1C_1$. Obično se podudarnost označava sa $\cong$.
Znači, ako su dva trougla podudarna onda je: $\triangle ABC \cong \triangle A_1B_1C_1 \rightarrow a = a_1, b = b_1, c = c_1, \alpha = \alpha_1, \beta = \beta_1, \gamma = \gamma_1$
Postoje 4 teoreme (stava) o podudarnosti trouglova:

<u>Stav SSS</u>

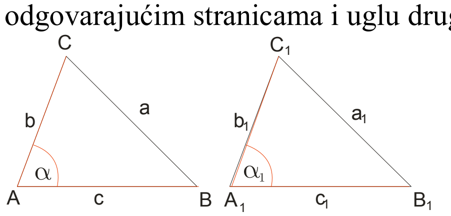
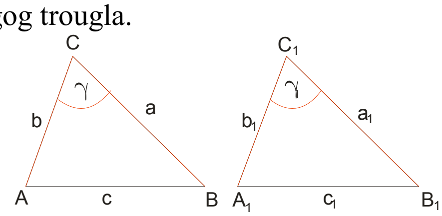
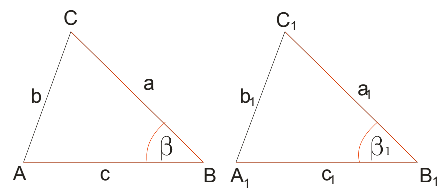
Dva trougla su podudarna ako i samo ako su stranice jednog trougla jednake odgovarajućim stranicama drugog trougla.



$\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow a = a_1 \wedge b = b_1 \wedge c = c_1$

<u>Stav SUS</u>

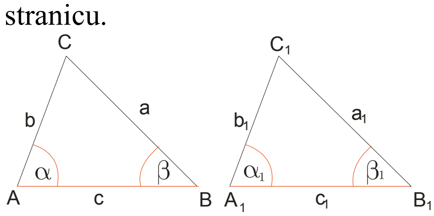
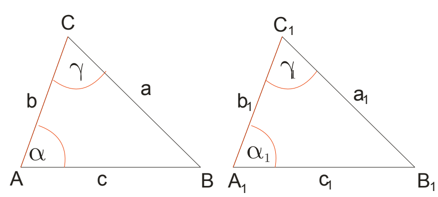
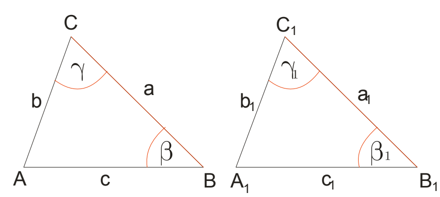
Dva trougla su podudarna ako i samo ako su dve stranice jednog trougla i ugao zahvaćen njima jednaki odgovarajućim stranicama i uglu drugog trougla.

$\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow b = b_1 \wedge c = c_1 \wedge \alpha = \alpha_1$ $\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow b = b_1 \wedge a = a_1 \wedge \gamma = \gamma_1$ $\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow a = a_1 \wedge c = c_1 \wedge \beta = \beta_1$

<u>Stav USU</u>

Dva trougla su podudarna ako i samo ako imaju jednaku po jednu stranicu i oba odgovarajuća ugla nalegla na tu stranicu.

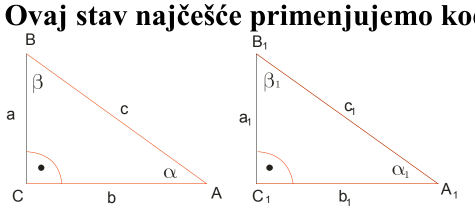
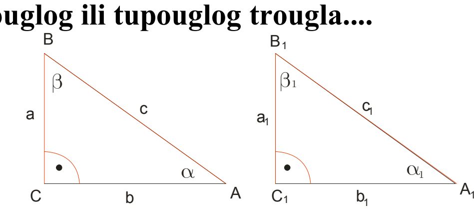




$\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow c = c_1 \wedge \beta = \beta_1 \wedge \alpha = \alpha_1$ $\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow b = b_1 \wedge \gamma = \gamma_1 \wedge \alpha = \alpha_1$ $\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow a = a_1 \wedge \beta = \beta_1 \wedge \gamma = \gamma_1$

<u>Stav SSU</u>

Dva trougla su podudarna ako i samo ako su dve stranice i ugao naspram jedne od njih u jednom trouglu jednaki sa dve odgovarajuće stranice i uglom u drugom trouglu, a uglovi naspram druge stranice u oba trougla su iste vrste (oba oštra ili oba prava ili oba tupa)

Ovaj stav najčešće primenjujemo kod pravouglog ili tupouglog trougla....

$\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow c = c_1 \wedge b = b_1 \wedge \gamma = \gamma_1 = 90^0$ $\triangle ABC \cong \triangle A_1B_1C_1 \Leftrightarrow c = c_1 \wedge a = a_1 \wedge \gamma = \gamma_1 = 90^0$

Primer 1.

Dokazati da su trouglovi $\triangle ABC$ i $\triangle A_1B_1C_1$ podudarni kada su im jednaki sledeći odgovarajući elementi:

a) $a = a_1, b = b_1, h_b = h_{b_1}$

b) $a = a_1, c = c_1, t_c = t_{c_1}$

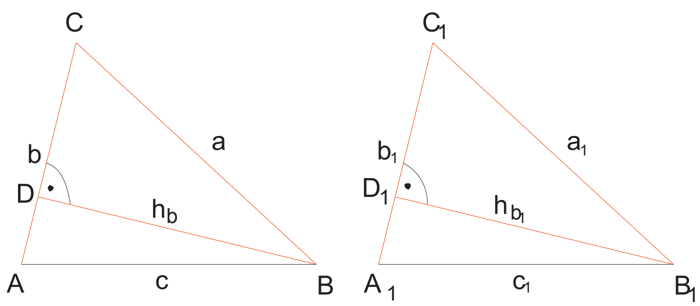
c) $c = c_1, t_c = t_{c_1}, h_c = h_{c_1}$

d) $\gamma = \gamma_1, b = b_1, s_\gamma = s_{\gamma_1}$

Rešenje:

a) $a = a_1, b = b_1, h_b = h_{b_1}$

Nacramo najpre sliku i na njoj drugom bojom (kod nas crvenom) obeležimo date jednake elemente!



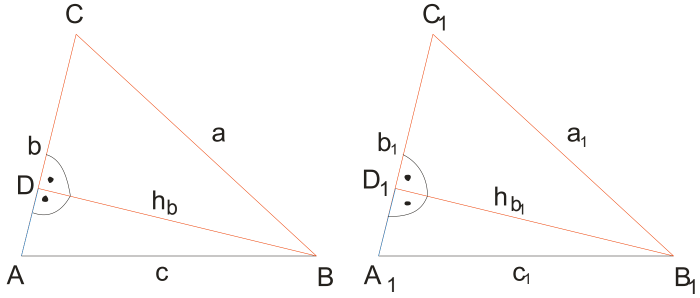
Uvek prvo dokazujemo za delove koji se “ više šarene” !

Dakle prvo dokazujemo da je $\triangle DBC \cong \triangle D_1B_1C_1$

Moramo da nadjemo tri elementa koja su jednaka I da kažemo koji je stav u pitanju!

$$\left.\begin{array}{l} a = a_1 \\ h_b = h_{b_1} \\ \measuredangle D = \measuredangle D_1 = 90^0 \end{array}\right\} \xrightarrow[SSU]{} \triangle DBC \cong \triangle D_1B_1C_1$$

E sad , odavde moramo izvesti neki zaključak koji će nam pomoći da dokažemo da je $\triangle DBA \cong \triangle D_1B_1A_1$



Ovde je taj zaključak da je $AD = A_1D_1$ jer je $AC = A_1C_1$ dato u zadatku a mi smo dokazali da je $DC = D_1C_1$

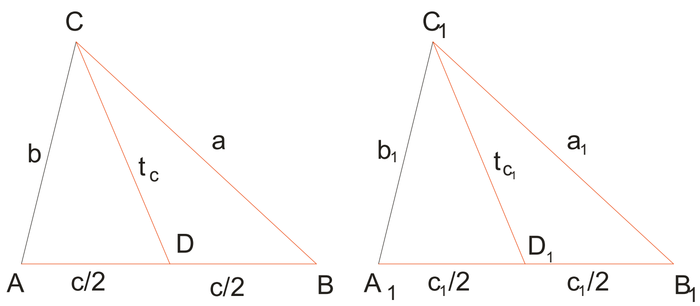
Sad možemo dokazati da je $\triangle DBA \cong \triangle D_1B_1A_1$

$$\left.\begin{array}{l} AD = A_1D_1 \\ h_b = h_{b_1} \\ \measuredangle D = \measuredangle D_1 = 90^0 \end{array}\right\} \xrightarrow[SUS]{} \triangle DBA \cong \triangle D_1B_1A_1$$

Iz svega sledi da je $\triangle ABC \cong \triangle A_1B_1C_1$

b) $a = a_1, c = c_1, t_c = t_{c_1}$

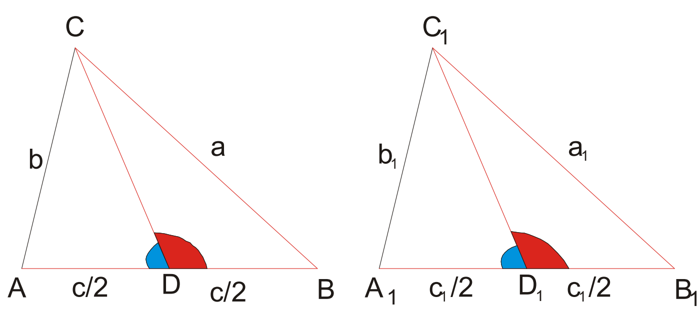
Dakle, nacrtamo sliku i ofarbamo zadate jednake elemente:



Sad krećemo od desnih trouglova:

$$\left.\begin{array}{l} a = a_1 \\ t_c = t_{c_1} \\ \dfrac{c}{2} = \dfrac{c_1}{2} \end{array}\right\} \xrightarrow[SSS]{} \triangle DBC \cong \triangle D_1B_1C_1$$

Da izvučemo zaključak koji nam treba za drugi deo dokaza:



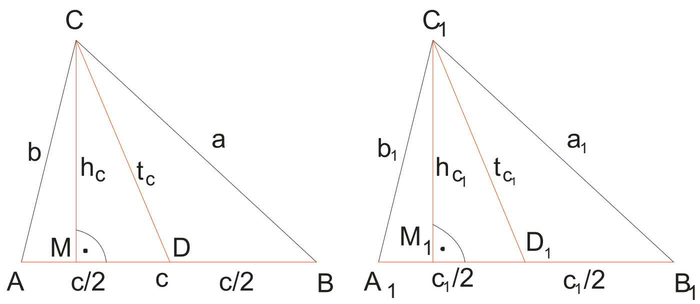
$\measuredangle BDC = \measuredangle B_1D_1C_1 \rightarrow \measuredangle ADC = \measuredangle A_1D_1C_1$

(Dopuna do opruženog ugla)

$$\left.\begin{array}{l} \measuredangle ADC = \measuredangle A_1D_1C_1 \\ t_c = t_{c_1} \\ \dfrac{c}{2} = \dfrac{c_1}{2} \end{array}\right\} \xrightarrow[SUS]{} \triangle DAC \cong \triangle D_1A_1C_1$$

Iz svega sledi da je $\triangle ABC \cong \triangle A_1B_1C_1$

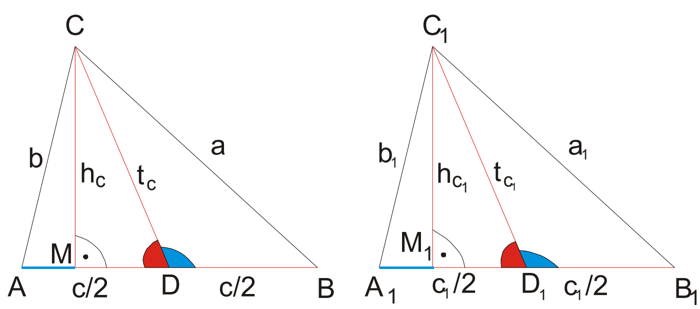
c) $c = c_1, t_c = t_{c_1}, h_c = h_{c_1}$



Pazite, ovde dokaz moramo izvesti za sva tri trougla. Krećemo od srednjeg….

$$\left.\begin{array}{l} t_c = t_{c_1} \\ h_c = h_{c_1} \\ \measuredangle M = \measuredangle M_1 = 90^0 \end{array}\right\} \underset{SSU}{\rightarrow} \triangle DMC \cong \triangle D_1M_1C_1$$

Ovde moramo izvući dva zaključka:



$\measuredangle MDC = \measuredangle M_1D_1C_1 \rightarrow \measuredangle BDC = \measuredangle B_1D_1C_1$

(ovo nam treba za desni trougao)

$MD = M_1D_1 \rightarrow MA = M_1A_1$

(ovo nam treba za levi trougao)

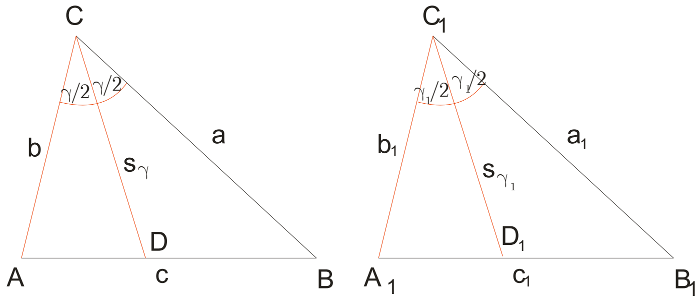
$$\left.\begin{array}{l} MA = M_1A_1 \\ h_c = h_{c_1} \\ \measuredangle M = \measuredangle M_1 = 90^0 \end{array}\right\} \underset{SUS}{\rightarrow} \triangle AMC \cong \triangle A_1M_1C_1$$ **(za levi trougao)**

$$\left.\begin{array}{l} \frac{c}{2} = \frac{c_1}{2} \\ t_c = t_{c_1} \\ \measuredangle BDC = \measuredangle B_1D_1C_1 \end{array}\right\} \underset{SUS}{\rightarrow} \triangle DBC \cong \triangle D_1B_1C_1$$ **(za desni trougao)**

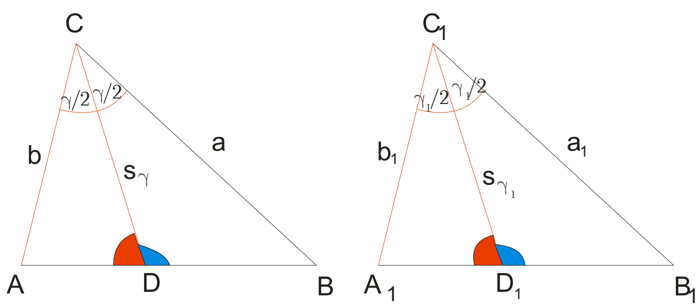
Iz svega sledi da je $\triangle ABC \cong \triangle A_1B_1C_1$

d) $\gamma = \gamma_1, b = b_1, s_\gamma = s_{\gamma_1}$

Da se podsetimo s_γ **je dužina simetrale ugla** γ **(deli ugao na dva jednaka dela)**



$$\left.\begin{array}{l} \dfrac{\gamma}{2} = \dfrac{\gamma_1}{2} \\ s_\gamma = s_{\gamma_1} \\ b = b_1 \end{array}\right\} \xrightarrow[SUS]{} \triangle DAC \cong \triangle D_1A_1C_1$$



Zaključak koji nam treba: $\measuredangle ADC = \measuredangle A_1D_1C_1 \rightarrow \measuredangle BDC = \measuredangle B_1D_1C_1$

Sad dokaz za desni trougao:

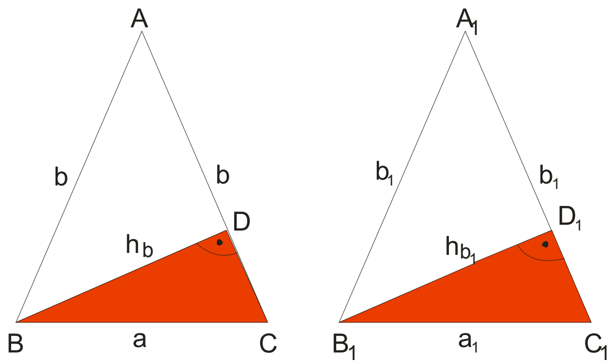
$$\left.\begin{array}{l} \dfrac{\gamma}{2} = \dfrac{\gamma_1}{2} \\ s_\gamma = s_{\gamma_1} \\ \measuredangle BDC = \measuredangle B_1D_1C_1 \end{array}\right\} \xrightarrow[USU]{} \triangle DBC \cong \triangle D_1B_1C_1$$

Iz svega sledi da je $\triangle ABC \cong \triangle A_1B_1C_1$

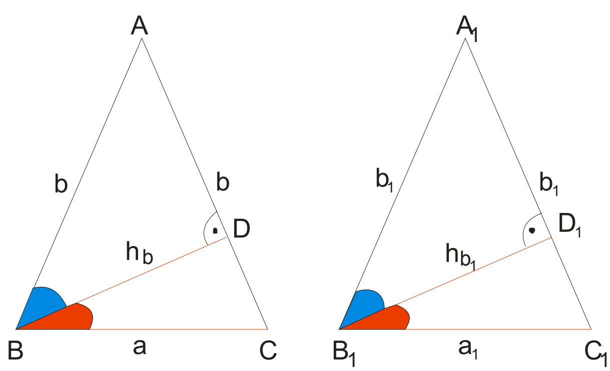
Primer 2.

Dokazati da su dva jednakokraka trougla podudarna ako su im jednaki elementi $a = a_1, h_b = h_{b_1}$

Rešenje:



$$\left.\begin{array}{l} a = a_1 \\ h_b = h_{b_1} \\ \measuredangle D = \measuredangle D_1 = 90^0 \end{array}\right\} \xrightarrow[SSU]{} \triangle DBC \cong \triangle D_1B_1C_1$$



Izvučemo zaključak iz prvog dela dokaza da je: $\measuredangle DBC = \measuredangle D_1B_1C_1 \rightarrow \measuredangle ABD = \measuredangle A_1B_1D_1$ **jer je početni trougao jednakokrak.**

$$\left.\begin{array}{l} \measuredangle ABD = \measuredangle A_1B_1D_1 \\ h_b = h_{b_1} \\ \measuredangle D = \measuredangle D_1 = 90^0 \end{array}\right\} \xrightarrow[USU]{} \triangle DBA \cong \triangle D_1B_1A_1$$

Iz svega sledi da je $\triangle ABC \cong \triangle A_1B_1C_1$

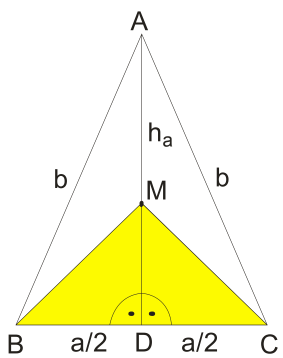
Primer 3.

Na visini AD koja odgovara osnovici BC jednakokrakog trougla ABC uočena je tačka M.

Dokazati da je MB = MC .

Rešenje:

Jednostavno uočimo dva trougla koji sadrže date duži i dokažemo da su oni podudarni a onda sledi da te duži moraju da budu jednake.



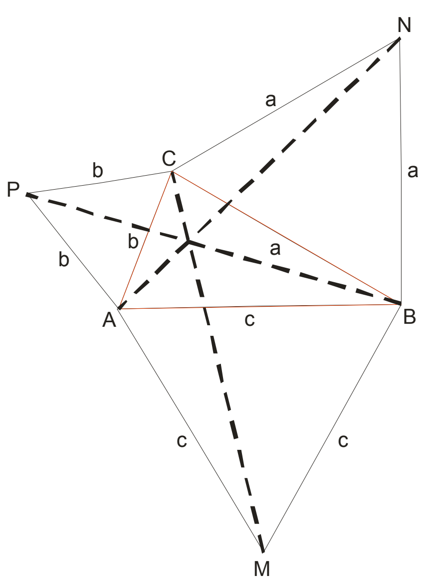
Uzećemo da je tačka M unutar trougla, a dokaz bi bio isti i da je na visini van trougla.

$$\left.\begin{array}{l} \frac{a}{2}=\frac{a}{2} \\ MD = MD \\ \measuredangle D = \measuredangle D_1 = 90^0 \end{array}\right\} \xrightarrow[SUS]{} \triangle DBM \cong \triangle D_1B_1M_1$$ **odavde sledi da je MB = MC**

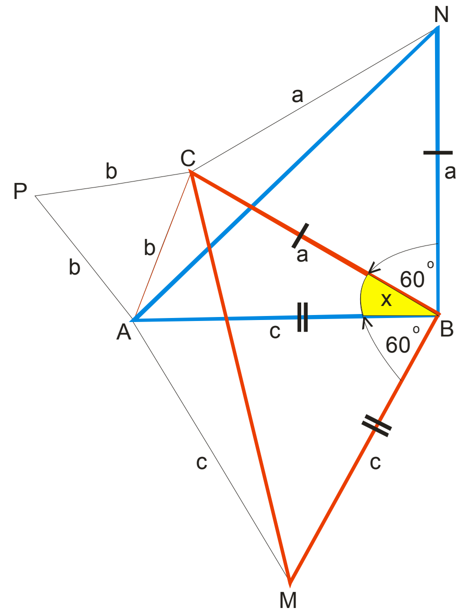
Primer 4.

Dat je trougao ABC. Na njegovim stranicama spolja konstruisani su jednakostranični trouglovi ABM, BCN i ACP. Dokazati da su duži AN, BP i CM jednake.

Rešenje:
Da nacratmo najpre sliku i postavimo problem:



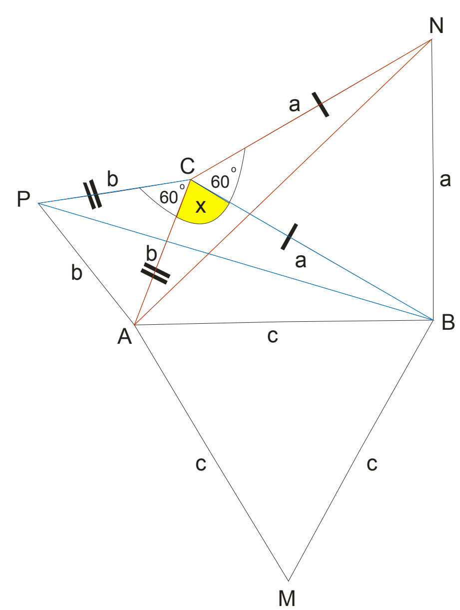
Uočimo trouglove koji sadrže duži CM i AN .



Dokazujemo da su trouglovi BCM i ABN podudarni (crveni i plavi na slici)

$$\left.\begin{array}{l} BC = BN = a \\ MB = AB = c \\ \measuredangle MBC = \measuredangle ABN = 60^0 + x \end{array}\right\} \underset{sus}{\rightarrow} \triangle BCM \cong \triangle ABN$$ **pa je odavde CM = AN**

Uočimo trouglove koji sadrže AN i PB . To su trouglovi ACN i BPC



$$\left.\begin{array}{l} BC = CN = a \\ CP = AC = b \\ \measuredangle BCP = \measuredangle NCA = 60^0 + x \end{array}\right\} \underset{sus}{\rightarrow} \triangle BCP \cong \triangle ACN$$ **pa je odavde AN = PB**

Iz svega sledi da su duži AN, BP i CM jednake !