

2. UČENIK UME DA KORISTI OSOBINE STEPENA I KVADRATNOG KORENA

Da se podsetimo još jednom pravila:

Za stepenovanje:

$$1) a^m \cdot a^n = a^{m+n}$$

$$2) a^m : a^n = a^{m-n}$$

$$3) (a^m)^n = a^{m \cdot n}$$

$$4) (a \cdot b)^n = a^n \cdot b^n$$

$$5) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Za korenovanje:

$$1) (\sqrt{a})^2 = a \quad a \geq 0$$

$$2) \sqrt{a^2} = |a|$$

$$3) \sqrt{a} \sqrt{b} = \sqrt{ab}$$

$$4) \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Primer 1.

$$\text{Izračunati: } \frac{3^5 \cdot 9^3}{27^2 \cdot 3} =$$

Rešenje:

$$\frac{3^5 \cdot 9^3}{27^2 \cdot 3} = \frac{3^5 \cdot (3^2)^3}{(3^3)^2 \cdot 3^1} = \frac{3^5 \cdot 3^6}{3^6 \cdot 3^1} = \frac{3^5}{3^1} = 3^{5-1} = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

Primer 2.

$$\text{Izračunati: } \frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3}$$

Rešenje:

$$\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} = \frac{x^{12} \cdot x^3 : x^5}{(x^{5-2})^3} = \frac{x^{12+3-5}}{(x^3)^3} = \frac{x^{10}}{x^9} = x^{10-9} = x^1 = x$$

Primer 3.

Izračunati: $\frac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}}$

Rešenje:

$$\frac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}} = \frac{3^{n+1+n+2}}{3^{2n+4}} = \frac{3^{2n+3}}{3^{2n+4}} = [\text{pazi na zagrade zbog minusa}] = 3^{(2n+3)-(2n+4)} = 3^{2n+3-2n-4} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

Primer 4.

Izračunati: $\frac{2^3 \cdot 4^2}{16^2 : 8} =$

Rešenje:

Pazi, i ovde ne možemo odmah upotrebiti pravila za stepenovanje jer osnove nisu iste!

Zato ćemo uraditi:

$$4 = 2^2, 16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4, 8 = 2 \cdot 2 \cdot 2 = 2^3$$

Vratimo se u zadatak:

$$\frac{2^3 \cdot 4^2}{16^2 : 8} = \frac{2^3 \cdot (2^2)^2}{(2^4)^2 : 2^3} = \frac{2^3 \cdot 2^4}{2^8 : 2^3} = \frac{2^{3+4}}{2^{8-3}} = \frac{2^7}{2^5} = 2^{7-5} = 2^2 = 4$$

Primer 5.

Izračunati: $2^5 \cdot \frac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \frac{16^2 \cdot 4^2}{2^3 \cdot 8^3} =$

Rešenje:

$$\begin{aligned} & 2^5 \cdot \frac{4^3 \cdot 8}{4^2 \cdot 2^6} - 2^3 \cdot \frac{16^2 \cdot 4^2}{2^3 \cdot 8^3} = \\ & 2^5 \cdot \frac{(2^2)^3 \cdot 2^3}{(2^2)^2 \cdot 2^6} - 2^3 \cdot \frac{(2^4)^2 \cdot 2^4}{2^3 \cdot (2^3)^3} = \\ & \frac{2^5 \cdot 2^6 \cdot 2^3}{1 \cdot 2^4 \cdot 2^6} - \frac{2^3 \cdot 2^8 \cdot 2^4}{1 \cdot 2^3 \cdot 2^9} = \\ & \frac{2^5 \cdot 2^3}{2^4} - \frac{2^8 \cdot 2^4}{2^9} = \\ & \frac{2^8}{2^4} - \frac{2^{12}}{2^9} = 2^4 - 2^3 = 16 - 8 = 8 \end{aligned}$$

Primer 6.

Izračunati $3^4 + 36^6 + 15^7 - 6^{12} - 3^7 \cdot 5^7 =$

Rešenje:

$$\begin{aligned}
& 3^4 + 36^6 + 15^7 - 6^{12} - 3^7 \cdot 5^7 = \\
& 3^4 + (6^2)^6 + 15^7 - 6^{12} - (3 \cdot 5)^7 = \\
& 3^4 + \overbrace{6^{12} + 15^7 - 6^{12} - 15^7} = (\text{potiru se}) \\
& = 3^4 = 81
\end{aligned}$$

Primer 7.

Izračunati:

A) $\frac{3^7 + 3^5}{3^7 - 3^5}$

B) $\frac{2^8 - 2^5}{2^8 + 2^5}$

Rešenje:

U ovom zadatku ćemo upotrebiti distributivni zakon, odnosno izvući ćemo “zajednički” ispred zagrade!

A) $\frac{3^7 + 3^5}{3^7 - 3^5} = \frac{3^5 \cdot 3^2 + 3^5}{3^5 \cdot 3^2 - 3^5} = \frac{3^5(3^2 + 1)}{3^5(3^2 - 1)} = \frac{9 + 1}{9 - 1} = \frac{10}{8} = \frac{5}{4}$

B) $\frac{2^8 - 2^5}{2^8 + 2^5} = \frac{2^5 \cdot 2^3 - 2^5}{2^5 \cdot 2^3 + 2^5} = \frac{2^5(2^3 - 1)}{2^5(2^3 + 1)} = \frac{8 - 1}{8 + 1} = \frac{7}{9}$

Primer 8.

Uprosti izraze:

A) $2\sqrt{2} + \sqrt{72} - 3\sqrt{8}$

B) $-3\sqrt{(-2)^2} + (2\sqrt{3})^2 - (4\sqrt{5})^2$

Rešenje:

A)

$$\begin{aligned}
& 2\sqrt{2} + \sqrt{72} - 3\sqrt{8} = \\
& 2\sqrt{2} + \sqrt{36 \cdot 2} - 3\sqrt{4 \cdot 2} = \\
& 2\sqrt{2} + 6\sqrt{2} - 3 \cdot 2\sqrt{2} = \\
& 2\sqrt{2} + \cancel{6\sqrt{2}} - \cancel{6\sqrt{2}} = \boxed{2\sqrt{2}}
\end{aligned}$$

B)

$$\begin{aligned}
& -3\sqrt{(-2)^2} + (2\sqrt{3})^2 - (4\sqrt{5})^2 = \\
& -3\sqrt{4} + 2^2 \sqrt{3}^2 - 4^2 \sqrt{5}^2 = \\
& -3 \cdot 2 + 4 \cdot 3 - 16 \cdot 5 = \\
& -6 + 12 - 80 = \boxed{-74}
\end{aligned}$$

Primer 9.

Uprosti izraze:

A) $\frac{3}{\sqrt{7}} - \sqrt{28}$

B) $3\sqrt{3} + \sqrt{108} - 4\sqrt{27}$

Rešenje:

$$\begin{aligned} \text{A) } \frac{3}{\sqrt{7}} - \sqrt{28} &= \frac{3}{\sqrt{7}} - \frac{\sqrt{7}}{\sqrt{7}} - \sqrt{4 \cdot 7} \\ &= \frac{3\sqrt{7}}{\sqrt{7}} - 2\sqrt{7} \\ &= \frac{3\sqrt{7} - 14\sqrt{7}}{7} = -\frac{11\sqrt{7}}{7} \end{aligned}$$

$$\begin{aligned} \text{B) } 3\sqrt{3} + \sqrt{108} - 4\sqrt{27} &= \\ 3\sqrt{3} + \sqrt{36 \cdot 3} - 4\sqrt{9 \cdot 3} &= \\ 3\sqrt{3} + 6\sqrt{3} - 4 \cdot 3\sqrt{3} &= \\ 3\sqrt{3} + 6\sqrt{3} - 12\sqrt{3} &= -3\sqrt{3} \end{aligned}$$

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